TECHNICAL PAPER
RAC-TP-265

JULY 1967

A Formulation of Ground Combat Missions in Mathematical Programming Form

by
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PREPARED FOR THE
DEPARTMENT OF THE ARMY
Contract No. DA 44-188-ARO-1

COMBAT ANALYSIS DEPARTMENT
TECHNICAL PAPER RAC-TP-265
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FOREWORD

The work described in this paper was a necessary part of a development of general methods for measuring the effectiveness of tactical command control systems. The results appear to have a wider application. They should be helpful to studies on incremental changes in weapons characteristics, force mix, organizations, and tactics, indeed any study that requires a systematic ordering of outputs from combat simulations to measure marginal changes in the performance of tactical missions.

It is hoped that this approach may lead to a better understanding of the combat effectiveness of combined-arms organizations.

P. H. Lowry
Head, Combat Analysis Department

ACKNOWLEDGMENTS

The author gratefully acknowledges the stimulation and encouragement of Richard Longmire. His detailed and incisive criticism contributed immeasurably to the clarity and cohesiveness of the presentation. The author also wishes to thank Joseph A. Bruner, Gerald P. Higgins, Stanford E. Hoffman, Howard C. Olson, Eugene P. Visco, Louis A. Walsh Jr., and Richard F. Zimmerman for their patience, forbearance, and helpful suggestions during lengthy discussions preceding the formulation and for their assistance in reviewing the drafts.

Completion of this document was expedited by the outstanding secretarial support of Joyce Anderson and Frances Jansen.

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A Formulation of Ground Combat Missions in Mathematical Programming Form

ABSTRACT

An attempt to develop general methods for measuring cost and effectiveness implications of adding automatic data processing to command control systems for ground combat required the development of techniques for marginal-effectiveness analysis. One necessary step for such analysis was the formulation of ground combat missions to permit measurement of marginal mission performance. Examination of typical combat missions identifies three dimensions: resources, time, and area controlled by a military force. Typical missions are related to a closed continuum of tactical postures ordered on the basis of relative potential energy and movement rate. Three classes of increasingly severe constraints are identified and associated with decreasing potential energy. Two objective functions are identified: maximization of rate of advance for high-energy postures and minimization of rate of resource expenditure for low-energy postures. The quantifiability of the three dimensions of mission space is examined, and difficulties in aggregating different classes of resources and terrain of varying tactical value are recognized. A measure for assessing the degree of control over an area by a military force is postulated and tested in a simple mathematical model. Relating the performance of a mix of military missions to combat effectiveness is discussed.

INTRODUCTION

Statement of the Problem

This paper assumes that (a) all systems, personnel, procedures, tactics, doctrine, and organization, which are inherently a part of a military unit, have value only insofar as they support the unit in achieving its mission(s); (b) the objective and quantitative measurement of the value of any element of the unit, to include command control information systems, requires the formulation of military missions in mathematical terms. The specific problem addressed by this paper is the formulation in mathematical programming terms of verbal statements of ground combat missions, in such a way that an objective function and a feasible solution space can be identified for each type of mission.

Discussion

The requirement to formulate ground combat missions in mathematical programming form arose from an effort to develop a more general method for cost-effectiveness analyses of military information systems at the tactical level. The cost effectiveness of such systems is not directly measurable because the effectiveness of the information system is not independent of the operating system being controlled. The value of the information provided will vary with the characteristics of the military unit using the information. These characteristics in turn depend on the weapons (and other materiel) contained in the military unit. Great increases in the speed and accuracy of information processing or in the completeness and validity (truth and relevance) of the information being processed do not necessarily result in significant improvements in military unit performance.[1,2] To perform analyses of the marginal effectiveness of information systems, one must be able to relate changes in information characteristics to changes in the performance of military units. Changes in data-processing characteristics may be related to the performance of military units in carrying out military missions by the following discrete steps:[3]

(a) Relate data-processing characteristics to performance of information system functions.

(b) Relate performance of information system functions to performance of the information system as a whole.

(c) Relate performance of the information system to results of combat.

(d) Relate results of combat to mission performance.

This paper addresses the fourth and final step in this process. Unless ground combat missions can be formulated in mathematical programming form, i.e., as objective functions within constraints, there can be no adequate objective quantitative measurement of mission performance.

This requires, however, a more rigorous examination of two fundamental questions:

(a) Can the combat missions of land forces be expressed in analytic form?

(b) If so, can such analytic expressions be related to quantities susceptible to measurement?

This paper begins with an examination of typical combat missions and identifies three dimensions, common to such missions, that appear to define a "mission space." A significant number of combat missions appear expressible in terms of the following dimensions: resources, time, and area controlled by a military force. Combat missions are related to the tactical postures of the forces. Typical mission statements and the associated postures are examined to determine the constraints imposed in terms of resources time, and area. It is concluded that missions transcending all postures can be expressed in terms of one of two objective functions: (a) maximize rate of advance or (b) minimize resource-expenditure rate.

The quantifiability of each of the postulated dimensions of "mission space" is examined. There is no conceptual difficulty in measuring time or resources available and expended. There may be some difficulty in defining the area controlled by a military force in terms of readily measurable quantities. Area of control may be expressible and measurable in terms of movement rates. More specifically, it is postulated that degree of control of area is defined by the ratio of movement rate in the face of the enemy to movement rate when no enemy is present.

Finally, a mission statement from WWII is analyzed to demonstrate the technique, and a method is suggested for aggregating mission performance into combat effectiveness.

MISSION SPACE

At field army level, combat missions are usually stated in rather broad terms, characteristic of "letters of instruction" as distinct from operations orders. Following are examples of such wording:

Deny control of a geographic area, its population, and its resources to an enemy.

Restore a prior situation as regards control of a geographic area, its population, and its resources.

Gain control of a designated geographic area, its population, and its resources.

Destroy a designated enemy force within geographic limits.

The accomplishment of such missions is related to national goals (not purely military) and hence is normally subject to additional constraints either explicit or implied. These constraints affect one or more of the following: the geographic area of the operation; the resources, which may be employed or expended to include forces, weapon systems (including nuclear weapons), and targets that may be attacked; and time, which may be restricted by prescribing maximum or minimum times for accomplishing the mission as appropriate.

At echelons below field army, missions are stated in operations orders or the even less formal fragmentary orders, and the wording is somewhat more

explicit. The following list shows typical examples of missions appropriate at army, corps, and division level.

Execute a retrograde movement within specified time, space, or force factors.

Operate as a covering force for some other (usually larger) force.

Defend a designated area, its population, or its resources.

Execute a counterattack.

Execute a breakthrough at a designated time in a designated area.

Seize and occupy a specified objective area, terrain feature, population or communication center, or resource.

Destroy a designated enemy force (usually within specified geographical limits).

Conduct a pursuit and/or an exploitation within a designated (stated or implied) area.

As in the case of the broad missions normally assigned to field armies, mission statements appropriate for corps and division are normally hedged about with additional constraints on the area of operations; on resource expenditure and availability, including weapons that may be employed; or on the time available for the mission.

Despite the apparent broad scope and proliferation of these operational missions, which involve commitment of ground forces, a much smaller number of independent variables appear to be common to all such missions. The first is application of military force for the control of areas and their populations and resources. If not only gaining control but also denying control to an enemy be included, then the dimension of area applies equally to the offense and the defense. Note that, in both cases, optimization of area of control means maximization, i.e., maximizing the area controlled by the friendly side. Constraints on area can be applied for either the offense, "Do not move forward of line ALPHA prior to 1000 hours (because shells or bombs will be falling)," or the defense, "Keep the enemy forward of line BRAVO until after 1400 hours."

Constraints on the second dimension common to mission statements, resources, can be applied in two different though related ways. The resources that the commander may employ for the accomplishment of his mission are always limited. A second constraint, usually less in absolute value, is typically stated or implied: the total resources that the commander may actually expend in the accomplishment of his mission. Resource expenditure is measured by the resources (personnel, weapons, materiel) required to restore a fighting force to the same level of effectiveness it had before commitment to the given mission. This constraint on resource expenditure is usually implied in the statement, "Maintain the integrity of the force."

The third dimension that runs through all missions is time. Occasionally a future time may be specified precisely; more often this dimension is used to bound the military problem. A maximum bound is established for the offense (e.g., "Take the objective no later than 0800 hours") and a minimum bound for the defense (e.g., "Hold at least until 1100 hours").

These three dimensions seem to define the space in which missions can be expressed. However, before going on to examine the applicability of optimizing an objective function within constraints, it is necessary to define more explicitly the purpose of such a restatement of military missions. Restatement in analytic form is for use by analysts who need to measure the degree of mission performance. It is not intended to be used by a commander in the field in phrasing his statement of mission, which might (in the proper context) be of the form "Git thar fustes' with the mostes'." The restatement is much more an effort to state in logical form the basis on which that same commander may compare the performance of two subordinates, each of whom receives the above order, but who actually arrive "thar" with varying degrees of "fustes'" and "mostes'." Psychological factors may well dominate physical factors or mere considerations of logic when enunciating orders, especially in the heat of battle. We shall attempt to perform the difficult task of restating, in analytic form, not what the commander said but what his logical purpose was when he said it.

A second caveat arises, almost as a corollary, from the purpose of this analysis of missions. Any attempt to order the goals of so involved a human activity as military conflict requires careful distinction between the relatively steady state or tactical posture associated with striving to achieve a given mission and the frequently violent changes in posture that can accompany transition to a new mission. Failure to make this distinction leads to such apparent anomalies as these: a classical transition from a defensive posture to the offense can begin with a partial withdrawal, followed by a counterattack leading to a breakout; or an effective method for going over to the defense begins with a spoiling attack. The purpose of this analysis is not to model the dynamics of conflict but to order the steady states.

One more notion is required to develop a general method for mapping a large body of ground force missions into statements of objective functions and constraints. It is possible to relate the bulk of combat missions more or less uniquely to their corresponding (steady-state) tactical postures and thus to refer to missions in such terms as meeting engagement, delaying action, covering mission, position defense, and penetration. Such a classification can be arranged in a closed continuum (see Fig. 1).

The natural evolution from any of the tactical postures identified around the periphery of the circle in Fig. 1 is to an adjacent posture. Jumping across one or more to get to the next succeeding posture is not precluded but is certainly rare. The arrangement of postures in a circle permits the identification of the right half as the offensive sector and the left half as the defensive sector. Note that offense merges into defense at the meeting engagement and again in going from a counterattack, normally associated with the defense, to a limited objective attack. Note also that the line that divides offense from defense separates in some sense the complementary postures or natural duals: delay is normal counter to pursuit, mobile defense to the envelopment, area defense to the penetration, etc.

In order to identify the analytic functions associated with each of these tactical postures, it is necessary to examine typical mission statements that could have resulted in these postures. The feasible solution space defined by

the mission statements is indicated in Table 1, in terms of constraints in each of the postulated dimensions of mission space. An entry "X" under a given dimension means that the mission statement does establish a constraint, either upper or lower as indicated. The entry "none" means that the mission statement leaves that dimension unconstrained in the indicated direction. The entry

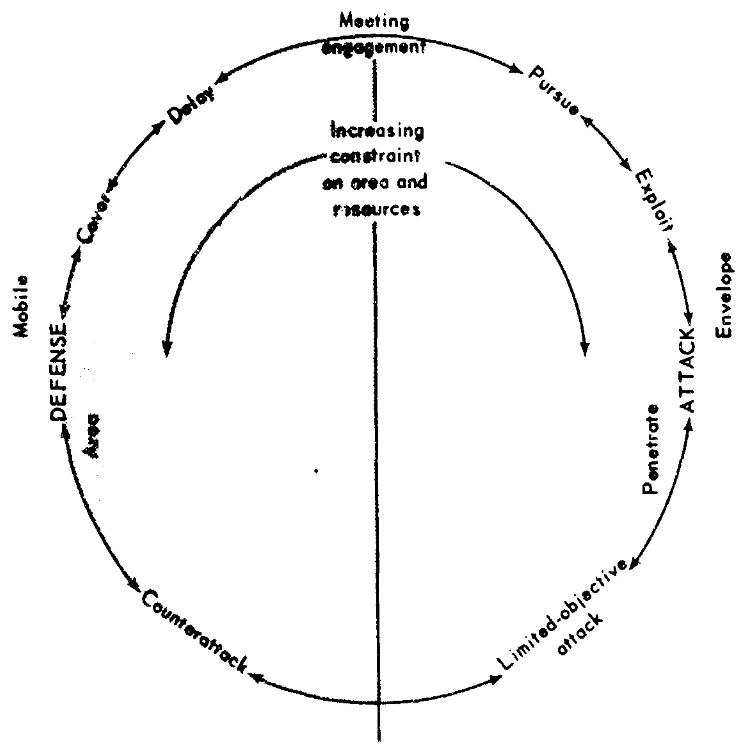

Fig. 1—Posture Continuum

"0", on the other hand, means that the constraint, either stated or implied, is actually zero. For example, the lower bound on resources is zero for all cases because the commander can use as few resources as he thinks he can get away with. Meeting engagement has been omitted because, strictly speaking, this is not a steady state but a transition from no conflict into one of the other postures—precisely which posture is determined by who got there "first" with the "most"—and employs it.

The first conclusion to be drawn from such a listing (Table 1) is that none of the mission statements identify the objective function. They only block out solution spaces in which solutions are to be found. These solution spaces fall into three classes (Table 2), where a class is defined to consist of all the missions subject to the same constraints. Note again that the duals identified in Fig. 1 always fall into the same class, as one would expect (the sense of the single constraint on area for pursuit and delay is obviously reversed).

TABLE 1

Feasible Solution Spaces

Posture	Mission statements	Limits	Constraints: Area	Constraints: Time	Constraints: Resources
Pursuit	Division attacks at (___) hours (from present positions) and destroys enemy in sector	Lower	×	×	0
		Upper	None	None	×
Exploitation	Division attacks at (___) hours passes through (____) advances along (____) axis seizes[a] (________)	Lower	×	×	0
		Upper	×[a]	None	×
Envelopment and penetration	Division attacks at (___) hours through (___), seizes crossings over (____) River at (____) and (____)	Lower	×	×	0
		Upper	×	None	×
Limited objective attack	Division attacks at (___) hours (from present positions), seizes Corps objectives 1 and 2 not later than (___) hours	Lower	×	×	0
		Upper	×	×	×
Counterattack	Division attacks (___) hours, destroys enemy in penetration, seizes high ground at (___), restores the battle area, and prepares to (______)	Lower	×	×	0
		Upper	×	×[b]	×
Area defense and mobile defense	Division defends without delay in sector (overlay) holding enemy forward of (overlay)	Lower	×	×	0
		Upper	×	None	×
Covering action	Division (as corps covering force) holds enemy north of line (___) until (___) hours, achieves maximum delay to provide adequate time for preparation of main corps defensive position, then corps striking force	Lower	×	×	0
		Upper	×	None	×
Delay	Division defends in sector (overlay) on successive positions, achieves maximum delay	Lower	None	×	0
		Upper	×	None	×

[a]The word "seize" is used here and in the succeeding mission statements in the restricted military sense of capturing the arbitrarily restricted portion of enemy-held territory identified in this order <u>and no more</u>.[4] It is in this sense that seize imposes an upper constraint on area.

[b]Although not explicitly stated, a maximum time for completion of a counterattack is implicit, since the enemy must be expelled before he can build up in the penetration to the extent where the mission is no longer feasible.

The most elementary choice of objective function would be to select for optimization the unconstrained dimension. But this immediately leads to a series of dilemmas: Class I missions would then have two objective functions

TABLE 2

Constraint Classes

Class	Postures	Constraints		
		Area	Time	Resources
I	Pursuit, delay	Unbounded in one direction	Unbounded in one direction	Completely bounded
II	Exploitation, envelopment, penetration, area defense, mobile defense, covering action	Completely bounded	Unbounded in one direction	Completely bounded
III	Limited-objective attack, counterattack	Completely bounded	Completely bounded	Completely bounded

(or the commander would have a choice of objective functions) and Class III missions none. Even worse, for Class II missions, which appear to leave time as the function to be optimized, the variable in question, time, cannot be independently optimized, i.e., speeding up or slowing up of time is beyond human capability. What is within the commander's capability is a change in the time at which a given objective is reached or a change in the resources expended at the time the objective is attained. Furthermore, even for Class III missions, which would on this basis be achieved to the same degree by any solution within the specified constraints, the nagging suspicion remains that there must be some basis for comparing mission accomplishment.

But the above argument has pointed the way toward a resolution of the difficulty. If time cannot be independently optimized, what can be done is to optimize either the rate of change of area (rate of advance) or the rate at which resources are expended. Both of these are logically possible objective functions for all three classes of constraints. The question remains, which function should be associated with which constraint class?

An answer to this question can be found by noting that not only does the number of constraints imposed increase from Class I to Class III, but the relative magnitude of the difference between the upper and lower bounds also decreases. For area, this implies that the maneuver room required to adopt a posture associated with constraint Class I is far greater than for Class III. The time dimension is not bounded in both directions until Class III. For resources, this fact is not quite so obvious because Table 1 has indicated that lower bounds on resources are always zero. In a theoretical sense, this is true because the commander is always free to use as few resources as he finds necessary to accomplish a given mission. In the real world, however, this ignores the dynamics of the situation. The commander has effective control only of those resources not yet engaged (his reserve, indirect-fire support,

air support). If we interpret resource constraints in this fashion, these constraints too get closer and closer together in moving from Class I to Class III.

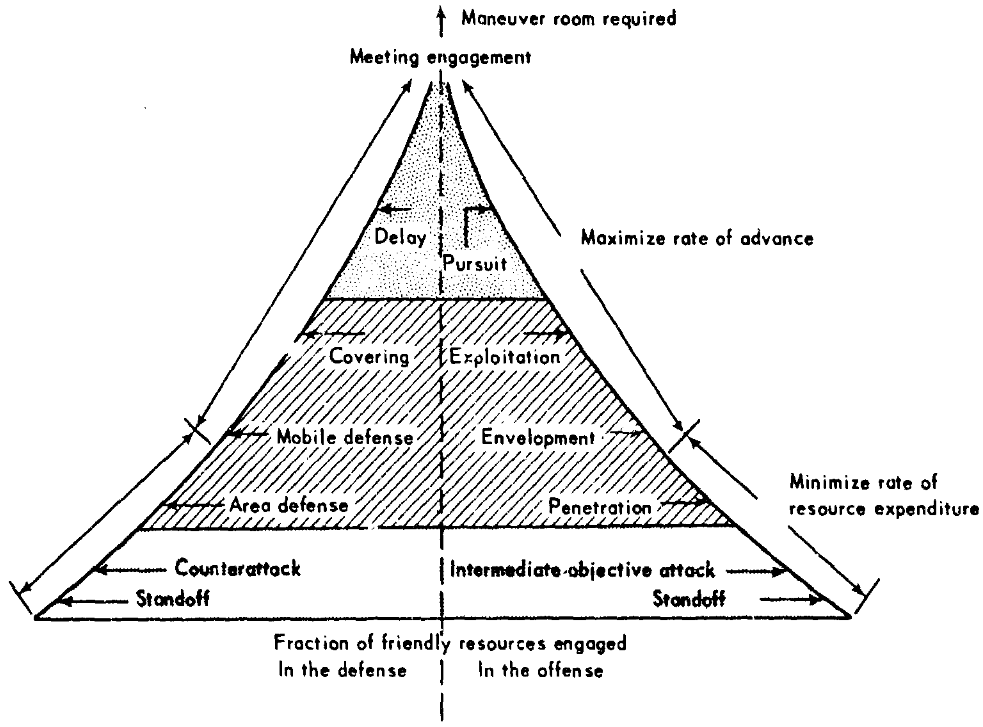

Fig. 2—A Taxonomy of Combat Postures, Classes of Constraints, and Missions

Constraint Class I   Constraint Class II   Constraint Class III

These qualitative observations have been plotted in Fig. 2. Along the X axis is plotted the fraction of the resources engaged—to the right for the offense and to the left for the defense. Along the Y axis is plotted the maneuver room required. The result preserves the sequence of postures plotted in circular form in Fig. 1, but they are now plotted along what can be interpreted as tradeoff curves between area and resources. Note the two extreme conditions. These are truly limiting conditions and are included for logical completeness. Whether they can exist in real life for any length of time is a moot point and does not really affect the basic argument. The first of these limiting conditions is the meeting engagement shown at the top in Fig. 2. This was previously described as a transient condition, once opposing forces are joined. At the outset, none of either force is actually engaged; hence the fraction of resources engaged is shown as zero. At the opposite extreme, if both forces have engaged their maximum resources and there is no transition to another state, an equilibrium must ensue; the result has been labeled "standoff."

Note also that the potential additional force that the commander has available (i.e., the fraction of the resources not engaged) increases from virtually zero at standoff to the total force at the meeting engagement. In a sense these

uncommitted resources represent potential energy, and thus the postures represent successively higher potential energy levels. Similarly, since the rate of advance is near maximum at the meeting engagement and approaches zero at standoff, the momentum must also be higher near the top of the diagram. (The natural evolution from one state to the next referred to in describing Fig. 1 is now seen to be a change of state requiring minimal change of energy.)

Since the ratio of available area to engaged resources is high near the top and low near the bottom, it is reasonable to associate maximization of rate of advance with the postures and missions near the top of the diagram and minimization of average resource-expenditure rate with those near the bottom. The point of transition is not readily identifiable in the abstract. It has been arbitrarily placed between envelopment and penetration on the attack side and between mobile and area defense on the defense side on the premise that, other things being equal, the envelopment and the mobile defense both require significantly more maneuver room and a higher fraction of initially uncommitted forces. Terrain can of course alter this premise significantly, and the distinctions between them are not always crystal clear. Also the exact location of the transition is not very important for the development of the analysis as long as one can conclude that at the upper extreme of Class II constraints the objective function is maximization of rate of advance and at the lower extreme it is minimization of resource-expenditure rate.

At this point one can very properly ask where to put the significant fraction of mission statements that seem to say, "Kill as many of the enemy as possible." In the context of the taxonomy of Fig. 2, one would have to say that such a statement is not truly an objective function but rather a tactic, a means to reduce the rate of expenditure of one's own resources (reduce friendly casualties) or to increase the rate of extending friendly control of area. And the most telling reason for subordinating maximization of enemy casualties seems to be that, although this objective is frequently sufficient to bring about a reduction in friendly resource expenditure or an increase in rate of advance, it is not always necessary for either.

QUANTIFIABILITY OF MISSION SPACE

Having determined a plausible set of dimensions for "mission space" and defined certain objective functions and constraints that permit mapping of a significant number of combat missions into analytic form, the question remains whether these variables are readily quantifiable.

One of the dimensions is time. As in most physical problems, time appears to be the independent variable par excellence. There are no conceptual difficulties in measuring times between events (e.g., changes in resources and in area), provided only that there are means for determining that those events have indeed occurred.

Quantity of resources was defined as a dimension of mission space and it was observed that constraints may be imposed both on the total available and the total to be expended. It has also been postulated that minimization of resource-expenditure rate is the objective function for a significant range of combat missions. Though resources may be difficult to measure, there are

no technical reasons that prevent measurement of resources. There is, however, another fundamental difficulty. What the foregoing analysis has defined is a resource function—not a method for aggregating resources. A solution within constraints that expends 50 lives and 10 tanks has minimized resource expenditure with respect to one that expends 50 lives and 20 tanks. But where is one ranked that expends 55 lives and 5 tanks? The difficulty is that "total resources expended" cannot be calculated until values are assigned to the various classes of resources. At this stage of the development we must depend either on the mission statement itself or the combat simulation to establish the necessary value structure. Fortunately, in a significant number of cases one class of resource, e.g., men, tends to dominate.

The third dimension was taken to be area controlled by military force. Maximum and minimum constraints were illustrated and rate of change of area (rate of advance) was postulated as an objective function for the remaining combat missions. For this dimension there is a twofold difficulty. Again, as for resources, we have really defined an area-of-control function—not a method for aggregating areas of widely different characteristics and hence military values. At this stage, this difficulty can be overcome only on a case-by-case basis, as was suggested for varying classes of resources. It is hoped that the mission statement will specify, or a combat simulation will demonstrate, the relative values of controlling different subareas. Again, we know from experience that the values of certain key terrain features may be so high as to dominate a much larger area.

Beyond this is a second difficulty that must be addressed before it is possible to make meaningful assessments of area controlled at a given time or of the rate of change of area being controlled. For this purpose a measure is needed that can be applied to each elementary area within a bounded geographic area of interest to assess the degree of control being exercised by each of the participants in land warfare.

A MEASURE FOR AREA OF CONTROL

The starting point for the development is the premise that absolute military control of an area by the friendly side means that there are no enemy-imposed constraints on friendly military operations in that area—the constraints that exist are physical or self-imposed. Any diminution from this ideal state of control represents less control by the friendly side and an increase of control by the enemy. The next step is, then, to select from among the myriad activities that comprise land warfare an activity or group of activities that is readily measurable and also sufficiently representative that measurement of enemy-imposed change in that activity can be used as a measure of degree of control. Since the primary concern is how the friendly area of control is increased or decreased, friendly movement rate seems to be a logical candidate. Actual rate of movement, even with no enemy present, is the result of an optimization process. If resource allocation is defined in a sufficiently broad sense, movement is essentially a process of minimizing resource expenditure (losses incurred by arriving too late vs losses resulting from traveling too fast, etc.). Presence of the enemy will in general result

in a different optimization choice, i.e., a different route or a different manner of moving may be chosen in an endeavor to minimize losses. Clearly then, enemy control implies the capability to alter the terms in the cost equation (developed later in this paper) and leads to a generally lower rate of advance for the same resource expenditure.

Optimum Velocity

Such a development can be initiated by considering the movement of a maneuver unit on the battlefield. Figure 3 is meant to illustrate such a unit,

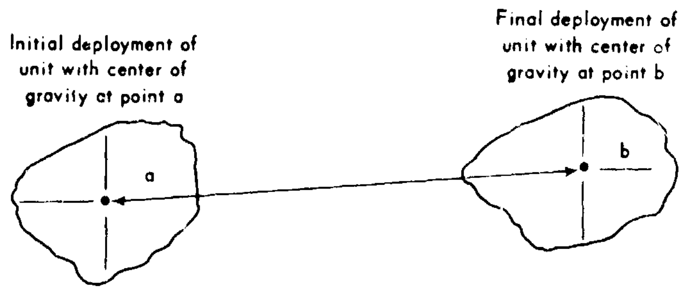

Fig. 3—Movement of a Maneuver Unit

with its elements deployed in a manner consistent with its tactical situation and mission but with its center of gravity initially at point a. This unit moves to a second position, where it is again deployed in some suitable manner but with its center of gravity now at point b. Assume that:

(a) The unit has moved distance s, in time t, at cost c measured in resources consumed and lost (destroyed).

(b) Any one move is made at uniform velocity v, i.e.,

$$v = s/t = \text{constant for one move.} \tag{1}$$

(c) The cost function is of the form

$$c = At + B(v)s \tag{2}$$

where A is a constant and B is a function of velocity.

There are two cases to examine to determine the effects on cost and distance of conducting such a move at different velocities.

<u>Case I</u>

Cost fixed at level C;
maximize distance s.

From Eqs 1 and 2,

$$C = A\frac{s}{v} + B(v)s,$$

$$s = \frac{Cv}{A + B(v)v}.$$

$$\frac{ds}{dv} = \frac{C[A + B(v)v] - Cv[B(v) + vB'(v)]}{[A + B(v)v]^2}; \text{ set } \frac{ds}{dv} = 0 \text{ to obtain } V_{opt}:$$

$$V_{opt} = \left[\frac{A}{B'(V_{opt})}\right]^{1/2}.$$

Case II

Distance fixed at S;

minimize c.

$$\frac{dc}{dv} = \frac{-SA}{v^2} + S\,B'(v) = 0 \text{ for minimum cost,}$$

and again,

$$V_{opt} = \left[\frac{A}{B'(V_{opt})}\right]^{1/2}. \qquad (3)$$

Assuming that $B(v)$ is a well-behaved function (monotonically increasing with continuous derivatives) as indicated in Fig. 4, plots of constant velocity

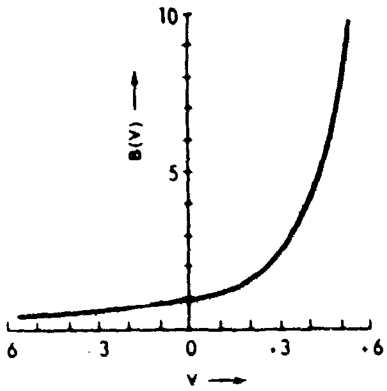

Fig. 4—Velocity-Dependent Component of Cost

and constant cost in s, t space can be made as illustrated in Fig. 5. As would be expected for such an elementary cost function and as indicated by Eq 3, the optimum velocity that simultaneously optimizes both distance and cost is uniquely defined.

The precise meaning of A and $B(v)$ is not so easily determined without further identification of some of the parameters associated with the movement. For example, assume personnel to be the critical resource in terms of which cost will be measured, and assume two essentially different casualty-producing mechanisms. One of these might be conceived as casualties resulting from some lethal agent (e.g., shell fragments, chemical agent, or radiation) that is assumed to be uniformly random in time and space and against which there is no cover. Then the unit will suffer casualties from this cause at a rate that

is a linear function of time, provided personnel are continuously replaced. The second mechanism results from the additional exposure to observed fire caused by movement. Figure 4 then denotes increased exposure for increased movement rates.

So far the development has provided no useful measures, but it raises some provocative questions. If such an optimum velocity is meaningful, then

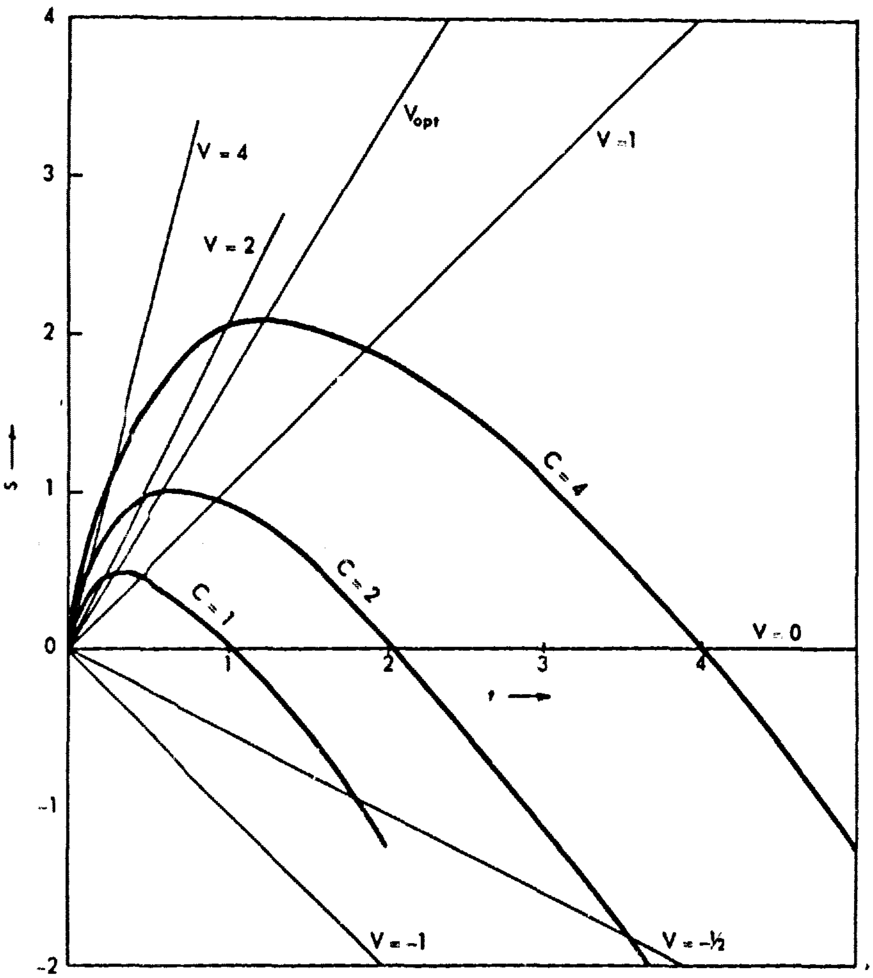

Fig. 5—Distance Moved as a Function of Time for Variations in Cost and Velocity

the ratio of that velocity in the presence of the enemy to velocity in the enemy's absence will vary with enemy capability and may be a measure of the control (e.g., of area) being exercised. This hypothesis is examined in the following section by applying it to movement on the battlefield in an environment that has been sufficiently simplified to permit constructing a simple mathematical model.

Movement under Fire

Since the purpose of the analysis is the definition of "control of area" in quantifiable terms, the effect of friendly fire and movement on enemy fire and movement and their interactions can be ignored without loss of rigor. Tempting as is the modeling of such interactions, it is essential to remember that a test probe is being designed with which to assess friendly control and enemy control without actually changing the degree of control exercised by either. This is no more farfetched a notion than measuring field strength without altering the field. In a sense a tactical commander does exactly this when he sends out patrols to assess the degree and distribution of enemy control without substantially altering the existing force-effectiveness ratio. It is therefore assumed that the space and time distribution of enemy fire is, for the purpose of this analysis, independent of the location of the test force.

Assume, on this basis, that one component of enemy fire is a flux that is uniformly distributed throughout the area of friendly movement and against which there is no cover. Call friendly exposure rate (hits per unit time) to this first component of enemy fire, e_1. Assume a second component, with higher peak values, that is caused primarily by enemy-observed fire weapons and to which friendly exposure occurs only during periods of actual movement. Call exposure rate to this second component, e_2. Experience indicates that maneuver elements (infantry and armor) move by bounds in such an environment to reduce exposure rate to the second type of fire. This is accomplished by moving at high speed (to increase enemy aiming error) in short spurts (to take advantage of enemy reaction time) to successive positions that afford cover and concealment (minimum exposure rate). Further simplification can be made in this model by assuming that cover and concealment against e_2 are uniformly and widely distributed so that the duration τ of the movement spurts is an independent variable against which exposure can be optimized. Under these conditions, movement can be represented as a series of ramp functions, as depicted in Fig. 6, that are generated by a series of moves at maximum speed r, and successive moves are interspersed with halts of duration t_s. From Fig. 6, it is clear that the aggregate distance covered, s, is given by

$$s = r \sum_{1}^{n} \tau \tag{4}$$

where n is the number of moves, and the average velocity v by

$$v = \frac{s}{t} = \frac{r \sum_{1}^{n} \tau}{t} . \tag{5}$$

Applying the simplification of uniform time intervals, it is noted that each bound covers distance $r\ \tau$, and that the number n of bounds required to cover distance s is

$$n = \frac{s}{r\tau} \tag{6}$$

and that

$$v = \frac{r\tau}{t_s + \tau} \tag{7}$$

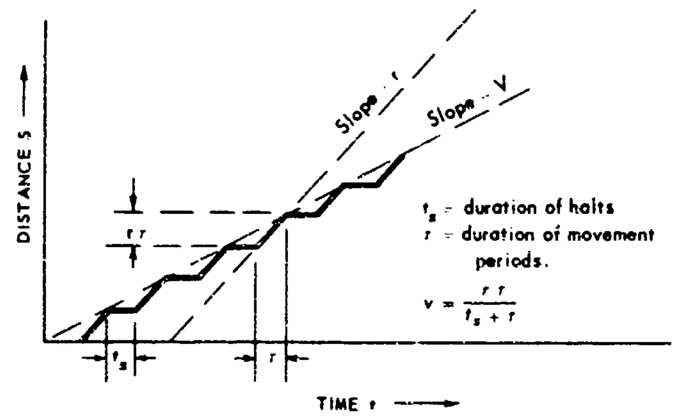

Fig. 6—Movement by Bounds

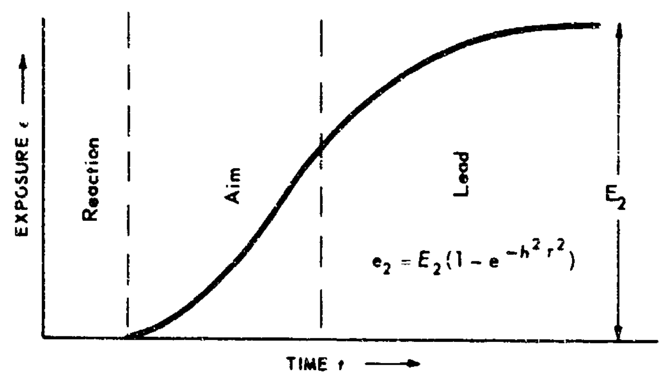

Fig. 7—Time Variation of Exposure to Direct Fire

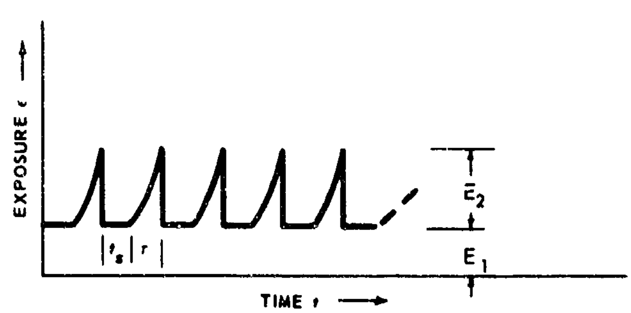

Fig. 8—Time Variation of Total Exposure

and

$$t = \frac{s}{r\tau}(t_s + \tau). \tag{8}$$

A cost function associated with such a movement can be defined. If cost is measured in casualties, exposure becomes a measure of cost. Assume that the number of hits, and therefore exposure, resulting from direct fire on a target that suddenly springs up and moves has a time variation of the form illustrated in Fig. 7. Exposure starts at zero because of enemy reaction time and rises rapidly as aiming error decreases but saturates at some value E_2 because of lead error and irreducible aiming error. Such a function can be approximated by the complement of the error function, i.e.,

$$\epsilon_2 = E_2(1 - e^{-h^2\tau^2}) \tag{9}$$

where h is a constant equal to the reciprocal of twice the variance of the error distribution. If the uniform flux of enemy indirect fire is superimposed, we add constant-exposure component $\epsilon_1 = E_1$, and the total exposure as a function of time looks like Fig. 8. Summing over the exposure time gives us the total exposure ϵ which is the total cost c for this example:

$$\epsilon = \int_0^t E_1\, dt + \sum_{i=1}^{n} \int_0^{\tau_i} E_2\left(1 - e^{-h^2\tau_i^2}\right) d\tau_i . \tag{10}$$

This general expression can be readily reduced for the simplifying assumptions of Eqs 6, 7, and 8 to

$$c = E_1 t + \frac{sE_2}{r} - \frac{sE_2}{r\tau}\int_0^{\tau} e^{-h^2\tau^2}\, d\tau . \tag{11}$$

Applying Eq 8 gives

$$c = \frac{s}{r}\left[E_1 + \frac{E_1 t_s}{\tau} + E_2 - \frac{E_2}{\tau}\int_0^{\tau} e^{-h^2\tau^2}\, d\tau\right]. \tag{12}$$

One other constraint must be applied before attempting to optimize this cost function. In Fig. 8 it can be seen that cost, as defined for this analysis, is essentially the area under the curve. This area can be reduced by reducing t_s. Bearing in mind that the cusps corresponding to time intervals τ are initial segments of the curve in Fig. 8, the area can further be reduced by shortening the duration of each movement time τ. This in turn necessitates increasing the number of moves. In the limit, there would be an infinity of such movement bounds, each of which would have a duration approaching zero. But there are real-world constraints that prevent optimization in this way. Not the least of these is that the duration of each static period, t_s, must equal or exceed some finite value if relation 9 is to hold. That is, the exposure to the observed fire component does not return to zero if the direct fire weapons have not returned to a static state. The further assumption is made that t_s has been reduced to the critical finite value required to ensure that $\epsilon_2 = 0$ at $\tau = 0$. This requires that there be a finite number of moves of duration τ.

If the derivative of the cost function defined by relation 12 is taken with respect to τ and the result is set equal to zero, the minimum cost relation becomes

$$\frac{E_1}{E_2} t_s = \int_0^{\tau} e^{-h^2 \tau^2} d\tau - \tau e^{-h^2 \tau^2} \qquad (13)$$

Making the usual series expansion of the error function, collecting terms, and simplifying yields the following relation, which is easier to handle for approximate calculations:

$$\frac{E_1}{E_2} t_s = \tau \left[2/3\, h^2 \tau^2 - 4/5 \left(\frac{h^2 \tau^2}{2!} \right)^2 + 6/7 \left(\frac{h^2 \tau^2}{3!} \right)^3 - \dots \right]. \qquad (14)$$

If a suitable range of values is selected for the parameters E_1, E_2, t_s, and h (which in turn fixes the standard deviation σ, since $h^2 = 1/2\sigma^2$), relation 14 can be used to calculate values of τ corresponding to minimum cost. Then relation 7 is used to calculate the corresponding ratios of velocity (at minimum cost) under fire, V_{opt}, to velocity r, which was assumed to be maximum velocity with no enemy opposition. Figure 9 shows how this ratio varies over a range of the other parameters.

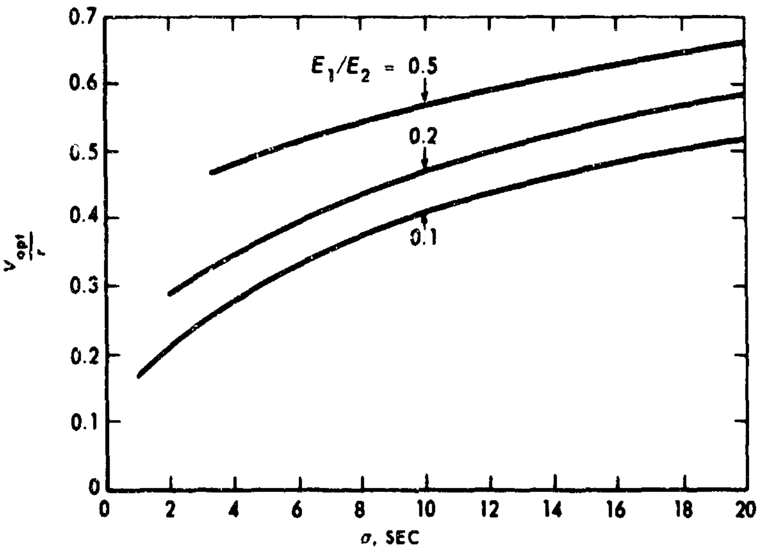

Fig. 9—Variation in Optimum Velocity Ratio Resulting from Enemy Fire

As expected, the above optimum rate-of-advance ratio increases as the observed fire exposure factor E_2 is reduced. This checks with reality; certainly the enemy has less control of the area through which friendly forces are moving if the intensity of enemy direct fire is reduced—or if friendly forces can provide armored protection so that they are in effect covered while moving. The ratio also increases as the σ of the inverse error function in relation 9 increases. This standard deviation is of course a measure of enemy-reaction time to friendly movement, and the more slowly the enemy reacts, the more rapidly friendly forces can advance.

Relations 13 and 14 reveal that the duration τ of the movement bounds becomes very large, and hence the optimum rate-of-advance ratio approaches unity, for either of two conditions:

(a) E_2 becomes very small. This case has already been discussed.

(b) E_1 becomes very large. This would imply that a high density of enemy-unobserved fire does not add to his control over area. Within the assumptions of this problem, this is true because it is known that a unit faced with the problem of moving through such a concentration of presumably unobserved fire does one of three things: moves through the fire at maximum velocity, waits until the fire lifts, or moves around the concentration. The terms of the analysis above permit description of only the first of the three alternatives, since it has been assumed that the unobserved fire flux is unbounded in space and time. The expression does give the right answer for these assumptions. In an actual case, such fires are of course bounded both in space and time, which permits one to wait it out or skirt the fire. Either of these alternatives slows the rate of advance, but the model would have to be extended to include them.

So far, then, the application of the ratio of optimum movement rates to an admittedly highly simplified model of movement under fire has led to no logical contradictions. It appears plausible that this ratio may be a useful measure of the degree of control being exercised over elementary areas of the battlefield. This measure is composed of quantifiable elements.

Use of Movement Rate Ratio as a Measure

If an attempt is made to use the optimum rate-of-advance ratio as a measure of the degree of control, careful examination must be made of some of its peculiar properties. As defined, it is only half a measure, in that only the degree of friendly control has thus far been defined, measured in terms of friendly rate-of-advance ratio. Presumably there should be a complement to this measure, i.e., one should be able to measure enemy degree of control of area by the enemy rate-of-advance ratio. Before looking into the nature of the relation between these complementary measures, one must be careful not to compare incommensurate quantities. As a result of expressing the actual rate of friendly advance on a per unit basis by dividing it by the maximum velocity of the same size unit with no enemy present, the ratio has been made very sensitive to the size of the military unit used to establish the measure. Figure 10 illustrates this in a qualitative way. Assume an isolated enemy battalion in position. If a path is judiciously chosen through this position, and certain minimal conditions of visibility prevail, it may be possible to move a single man or even a small patrol through the area at a velocity approaching their "peacetime" rate of movement. A force comparable in size to the enemy force would be discovered and suffer a severe degradation in its rate-of-advance ratio. On the other hand, a significantly larger friendly force tends more and more to approach its peacetime velocity, e.g., the optimum rate-of-advance ratio for a division, over normal terrain, would be much closer to unity than would be the ratio for the battalion or brigade moving through the enemy battalion. If this ratio is used as a measure of degree of control of area, one must always relate it to a specific unit. Furthermore, since the interactions between fire and movement have not been modeled and a dynamic situation is being examined in which both fire and movement are functions of time, this measure

will be valid for only a given instant. It can be used to draw isocontrol contours that are valid at a specified time.

On the other hand, because the rate of advance is expressed on a per unit basis related to the peacetime rate, it has been made insensitive to the multitude of other factors not enemy induced (e.g., terrain, weather) that would otherwise affect it.

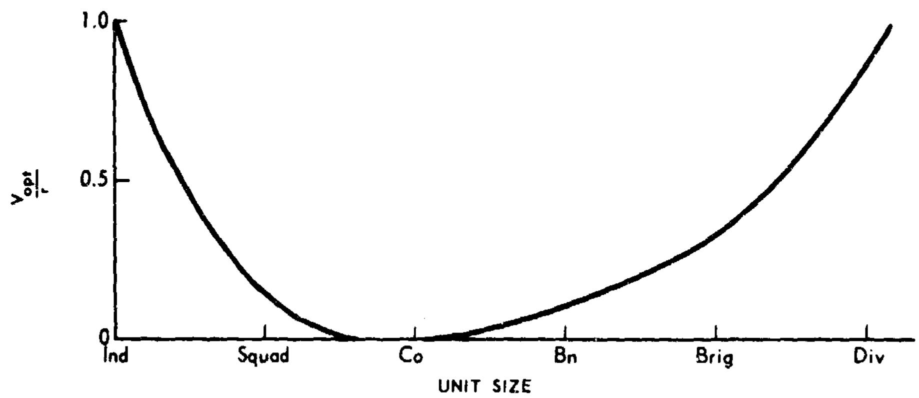

Fig. 10—Variation in Optimum Velocity Ratio with Unit Size

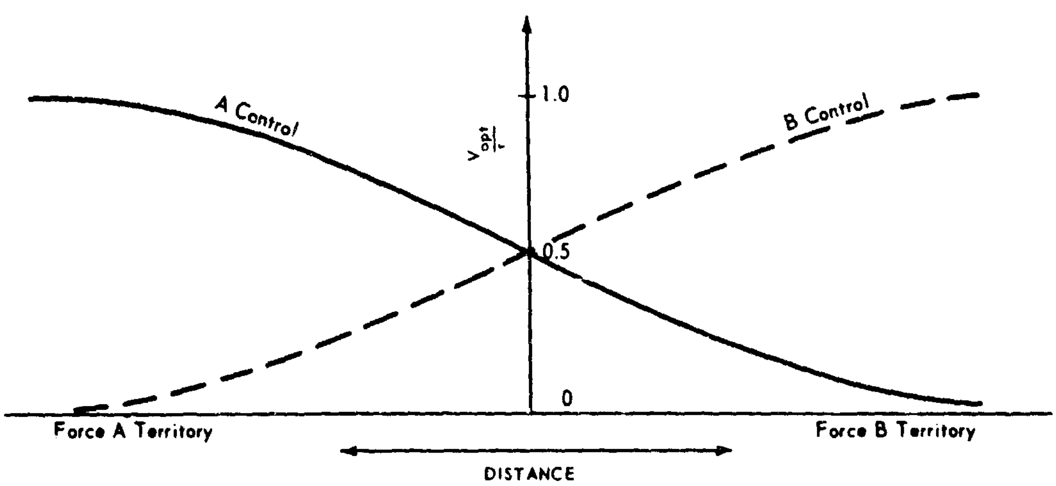

Fig. 11—Variation in Optimum Velocity along Section through FEBA

If determination has been made of the friendly and enemy forces that are to be considered in determining the degree of control exercised by each over the area in question at a specified time, this measure could be used to plot the isocontrol lines for each force. If a section of such a plot is inspected to see how the degree of control exercised by each varies as one moves a test probe from the area physically occupied by one force into the area occupied by the other, something like Fig. 11 may result. The interrelations between these two curves would be interesting to study. Is the value of one equal to 1 minus the

value of the other, etc.? Intuitively one speculates that the slope of such a curve is probably intimately related to the tactical posture. For example, for a position defense, one would expect it to be a very steep curve and for a withdrawal/exploitation, very shallow.

There may also be some relation between the depth of the military objective behind enemy lines and the crossover point on the curves that determines whether the objective function is minimization of resource-expenditure rate or maximization of rate of advance.

APPLICATION OF MISSION CONCEPTS

So far the development appears to have answered affirmatively the questions raised in the opening paragraphs. Three dimensions, time, resources, and area, do seem sufficient as dimensions for describing typical combat missions. Three distinct classes of constraint combinations and two distinct objective functions have been related to tactical postures that have in turn been related to typical mission statements. The postures have been arranged in a closed sequence determined by examining the relative magnitude of the maneuver area required and the fraction of resources committed. This formed the basis for selecting the objective function. The quantifiability of the dimensions used for structuring mission space has been examined and methods proposed for relating these dimensions to quantities that are measurable outputs of combat models. The fundamental question of how to aggregate dissimilar resources and areas has been recognized and is believed to be answerable only in the context of specific examples.

A remaining task is the application of this body of concepts to a specific combat mission—not for the purpose of proving the generality of concept but simply as a guide to its application. For this purpose it is advantageous to select a mission statement that is reasonably complete in itself, so that a minimum of intent can be inferred from the situation for which it was formulated. The following is a statement of the primary mission assigned to the 101st Abn Div for the Normandy invasion in June 1944:[5]

> 3. a. 101st A. B DIV (less certain glider elements) with Co D, 70th Tk Bn, 65th Armd FA Bn and Tr C, 4th Cav Sq, attached—BRIGADIER GENERAL MAXWELL D. TAYLOR, commanding.
>
> (1) Will land by parachute and glider at H-5 hours on D Day southeast of STE MERE EGLISE (3196) with the principal mission of assisting the 4th Div landing by seizing the western exits of the inundated area west of UTAH BEACH between ST. MARTIN DE VARREVILLE (4098) and POUPPEVILLE (4393), both inclusive [by H+1 hour and 30 minutes. However, no occupation of the high ground or seizure of these exits will occur prior to H-5 minutes.]*

The first step in transforming this mission into analytic form is to fill in the matrix of constraints as shown in Table 3.

In terms of the constraint classes identified in Table 3 the above mission falls into Class III, since the mission defines a feasible solution space that is

*Words in brackets do not appear in this paragraph but must be inferred from the operation schedule, specifically the Air Support Plan, Annex 5.

completely bounded in all three dimensions. The taxonomy of combat missions depicted in Fig. 2 provides the information that for missions of constraint Class III the objective function is minimization of resource expenditure. It would of course be rash to argue that this was necessarily the purpose in the mind of the Supreme Commander when this mission was assigned. On the other hand, it does provide a logical basis for comparing a series of trials (obviously

TABLE 3

Constraints on Area, Time, and Resources

Limit	Area	Time	Resources
Lower	Not explicitly stated, but can be inferred to be the initial defense perimeters around landing zones since this mission begins on completion of an air drop	H - 5 min	0
Upper	Western exits of the four causeways and the high ground overlooking these exits	H + 1 hr 30 min	The assault elements of the division; the magnitude of these elements was ultimately constrained by the aircraft available for the airlift

simulations) each of which did achieve a solution within the constraints specified in Table 3 but which differed in the time at which control of the exits was achieved and in the resources expended in achieving this control. The restatement in analytic form tells us that the series should be ranked on the basis of average resource-expenditure rate—not rate of advance—for all trials that did in fact fall within the specified constraints. For this example the critical resource that would probably dominate all others is trained manpower. Thus, casualty rate would be the applicable measure.

MISSION PERFORMANCE AND COMBAT EFFECTIVENESS

Although the preceding paragraphs have developed a taxonomy of combat missions that makes it possible to relate a large number of combat missions to one of two objective functions within one of three classes of constraints, conceptually a still higher qualitative measure is frequently used to describe the performance of military forces, i.e., combat effectiveness. This measure is rarely defined with sufficient precision to lend itself to quantification. Such a measure could, however, be related to mission performance somewhat as follows.

It can at least be argued that optimization of a military force for missions having constraints in Class III does not necessarily provide optimization for

missions having their constraints in the other classes in terms of such qualities as characteristics of its command control structure, firepower, and mobility. Note, for example, the rather obvious differences in a WWI division optimized pretty much within Class III constraints, an Afrika Korps division apparently optimized within Class II constraints in another environment, and a guerrilla force pretty much optimized within Class I constraints. With this line of reasoning, it is possible to define combat effectiveness in a quantifiable manner by defining a range of missions and environments for which the combat effectiveness of a given force is to be optimized and then assigning relative weights to the performance of each mission in each environment. From such a definition it is possible to define a composite objective function within specified constraints, and it is then possible to measure combat effectiveness in terms of a weighted set of mission performances.

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UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Research Analysis Corporation McLean, Virginia 22101	2a. REPORT SECURITY CLASSIFICATION Unclassified
	2b. GROUP Unclassified

3. REPORT TITLE

A FORMULATION OF GROUND COMBAT MISSIONS IN MATHEMATICAL PROGRAMMING FORM

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Paper

5. AUTHOR(S) (First name, middle initial, last name)

Roland V. Tiede

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
	25 + vi	15

8a. CONTRACT OR GRANT NO. DD 44-188-ARO-1	9a. ORIGINATOR'S REPORT NUMBER(S) RAC-TP-265
b. PROJECT NO. 007.402 WY 1967	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Automatic Data Field Systems Command Ft. Belvoir, Virginia

13. ABSTRACT

An attempt to develop general methods for measuring cost and effectiveness implications of adding automatic data processing to command control systems for ground combat required the development of techniques for marginal-effectiveness analysis. One necessary step for such analysis was the formulation of ground combat missions to permit measurement of marginal mission performance. Examination of typical combat missions identifies three dimensions: resources, time, and area controlled by a military force. Typical missions are related to a closed continuum of tactical postures ordered on the basis of relative potential energy and movement rate. Three classes of increasingly severe constraints are identified and associated with decreasing potential energy. Two objective functions are identified: maximization of rate of advance for high-energy postures and minimization of rate of resource expenditure for low-energy postures. The quantifiability of the three dimensions of mission space is examined, and difficulties in aggregating different classes of resources and terrain of varying tactical value are recognized. A measure for assessing the degree of control over an area by a military force is postulated and tested in a simple mathematical model. Relating the performance of a mix of military missions to combat effectiveness is discussed.

DD FORM 1 NOV 65 1473

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
ADSAF - TOS						
Combat Effectiveness						
Command Control						
Effectiveness Measures						
Information Systems						
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